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Towards the Horizon of Meaning: Interview with Partha Sarathi Chakraborty

Partha Sarathi Chakraborty at a restaurant. Partha Sarathi Chakraborty

Partha Sarathi Chakraborty is a mathematician at the Indian Statistical Institute (ISI), Kolkata. His research interests lie in operator algebras and noncommutative geometry. After completing his PhD under the supervision of Kalyan Bidhan Sinha at ISI Delhi, he held research positions at IHÉS and the Institute of Mathematical Sciences (IMSc), Chennai. In 2018, he joined ISI Kolkata as a Professor of Mathematics.

In this conversation with Saraswata Sensarma, Purnima Tiwari, and Aayush Verma for *Anveshanā*, Prof. Chakraborty reflects on his upbringing in Bengal, his early fascination with science, the meaning of doing mathematics, his approach to research and teaching, the influence of K. R. Parthasarathy's mentorship, and his journey through the different campuses of ISI—both as a mathematician and as an individual.

We are at this great institution for mathematics, speaking with a mathematician. So let us begin by asking: How has your understanding of mathematics evolved over the years, and what does it mean to you today?

Partha Sarathi Chakraborty: The meaning of mathematics for me has, of course, evolved with time; it had to.

As a kid, there was fun and a sense of power. You prove something, and then it becomes hard to refute. Nobody can challenge you, so it seems like the ultimate truth. There is a certain attraction to that. Then, of course, one experiences the fun of solving many problems, especially if one can solve them very quickly. It goes on for a while like that.

But then, as one grows, it evolves. When I entered the Indian Statistical Institute (ISI), there were many people, including some of my teachers, who have now retired, and who were very interested in mathematical logic. Under their influence, I also started studying logic, and then I realized that things are not really "true" in an absolute sense. That initial amazement diminished a little, but mathematics still remained immensely powerful.

On a train. PARTHA SARATHI CHAKRABORTY

Now, at the present juncture, when everybody is talking about AI and LLMs, there is a new task for mathematicians. Since all these depend on mathematics, we must point out its limitations. We must make it clear that proofs do not deliver absolute truth in the sense people often believe. We are essentially being taught not to question an algorithm's output, and that's a dangerous idea. While others may treat a proof as unquestionable, it is important to emphasize that we can—and should—question the output of an algorithm.

Additionally, one can now prove theorems using AI. Therefore, all of us should sit down and think: what does it mean to do mathematics? It is a very relevant question today. We see that almost every other day, one of Erdős' problems is being solved. There ought to be a collective discussion about what it means to say that we are doing mathematics.

Can you recall a mathematical problem, theorem, or idea that particularly fascinated you or changed the way you thought about mathematics?

"...proofs do not deliver absolute truth in the sense people often believe."

PSC: Even before my undergraduate studies, maybe in the 9th standard, I struggled quite a bit with a problem, though it was not very surprising. I still ask school kids this problem: consider a triangle, and then take the bisectors of the two angles at the base. If they have the same length, then show that the triangle is isosceles.[1] Generally, other geometry

[1]This is called the Steiner–Lehmus Theorem.

problems are very quick, but for this one, I really had to think for more than a month, which I enjoyed at the time.

At the undergraduate level, too, there was one surprising thing. In our first calculus class, our teacher wanted to define the natural numbers for us, so he said they are the "smallest" inductive set. Now, I got completely stuck, because the smallest among which collection? I searched a lot, and in those days, you did not have Google or the internet; you had to physically search for books in the library. I then learned that you can make this statement precise. I think Bernays, von Neumann, and Gödel, in their formulation of set theory, make a precise statement that the natural numbers are the smallest inductive set. This formulation is there in Dugundji's book. The statement was not really counterintuitive, because whatever inductive set you can think of, indeed, it looks like the natural numbers are the smallest, because you have to begin somewhere. Then you keep adding one, and you obtain the naturals. It was more about how to make the statement precise, and that requires a precise formulation of set theory.

Do you think mathematical maturity changes one's sense of what is obvious or counterintuitive?

PSC: Of course, as you pass to measure theory, various counterintuitive things start happening. But I do not remember anything off the top of my head. There are various theorems which are amazing, but I cannot say that they are counterintuitive. I think the sense that a theorem is obvious comes with extensive training and mathematical maturity. In general, things are not really obvious. For that, one has to become somewhat old and spend some time in an area; then you start seeing that some things are immediate.

And of course, counterintuitive things do come, but generally at a later stage. As you try to formulate a theory, maybe there are junctures where you do not have many examples to guide you through that development. It happens more in these quantum or non-commutative theories, where quite often you are guided by classical intuition. Now you ask yourself—what am I looking for? Am I looking for a problem or a development that parallels the classical development? Or something else that you do not see in the classical world?

If your intuition is guided too much by the classical world, then you do not find that really appealing. You may prove a result with a lot of hard work, but that is what you had expected to begin with. But say, even after hard work, you see that you are not getting what happens in the classical part, and you get something different; then it looks counterintuitive. I do not think it is a good idea to emulate the classical world too closely.

"If your intuition is guided too much by the classical world, then you do not find that really appealing."

Could you tell us a bit about your childhood and the environment you grew up in?

PSC: Just after my birth, we went to Jagdalpur and then moved to Shillong. I do have memories of Shillong, which was a very nice place, but none from my time in Jagdalpur. After a while in Shillong, however, we started experiencing political disturbances, so we moved to West Bengal. All of my early schooling happened in Bengal itself.

I would say that life in Bengal was very routine. At that time, it was very common for every child to do two things: study and learn to play some music. You would wake up in the morning, go to school, come back, and then practise music. Along with these activities, you had to study at a fixed time every day. I used to learn the sitar and practised it every day. It was a very rigid schedule, and there was no escaping it.

I vividly remember sitting with our books and studying even during Durga Puja. After studying, I would ask my mother if I could go pandal-hopping and participate in the festivities. So life was very routine, and having grown up like that, I still enjoy following a routine.

It seems that discipline and routine have remained an important part of your life.

PSC: Now that I wake up in the morning, I do certain things in a precise order, and it helps. I saw one of my professors, K.R. Parthasarathy (KRP), also following such a routine. But I must admit that I cannot follow a routine like his. Before his lectures, even when he knew exactly what he was going to say, he would still write everything down meticulously and note every point to be made. He maintained all his notebooks very carefully. I do write, but I do not maintain notebooks in quite the same way as he did.

K. R. Parthasarathy in 1975.
Baroc through Wikimedia Commons

The last photograph of KRP is probably in that interview of Anveshanā with Raja-da [B.V. Rajarama Bhat].[2] At that time, he was almost blind. Even then, I think he was proofreading his last book. His wife used to read it to him, and he would make corrections. He maintained that routine even at that stage of his life. I find that very inspiring.

How did you first come to meet K.R. Parthasarathy, and what do you remember about that first meeting?

PSC: I met him in 1997, just as I joined my PhD at ISI Delhi. I vividly remember meeting him for the first time. At that time, the ISI PhD entrance

[2]A Life in Hilbert Space: Interview with B.V. Rajarama Bhat, *Anveshanā e-Magazine*, January 2025, available at: https://anveshanamagazine.github.io/bv-rajarama-bhat.html

examination was very different from most other PhD entrance examinations. They used to ask four or five questions. Each question was actually a theorem, and they would develop the result through a series of lemmas. Even solving two questions was considered very good, but each problem had five or six parts, sometimes even more.

I wrote both the ISI test and the TIFR entrance examination. I was really not sure whether to go abroad or stay in India, but by the final stage, I decided to stay in India.

Every day, I would come from my home to the institute and wait outside the Dean's office, where they used to publish the results on the notice board. I would check the board and then go home. If ISI had published its results, I would not have gone to TIFR. But ISI did not publish the results of its JRF written test until quite late, so I had to join TIFR.

One day, I met the Dean and asked him about the entrance test results. He said, "Oh, I was looking for you all over the place. You are the only student to have cleared the ISI test. Where would you like your interview to be held?"

> *"He maintained that routine even at that stage of his life."*

I said, "Now it is too late because I have to go to TIFR the day after tomorrow. So I cannot have the interview right now." But after thinking for a bit, I requested that the interview be conducted in Kolkata. The Dean said, "No, you have to go to Delhi because they wish to test you there." So I joined TIFR, and from there I went to Delhi for the interview. That was a very memorable experience. That interview was my first encounter with KRP, and I still remember his first question.

What was it?

PSC: "What is the meaning of Chakraborty?" I said, "Chakra means a circle—the king's circle—and it refers to the person who is present in that circle." KRP said, "No, no. The circle is incomplete without him. He is the king. Chakraborty means Raj Chakraborty."

I actually remember the entire interview. Then he asked, "Can you give me an example of a stochastic process?" All my teachers at ISI Kolkata had already taught me that whenever somebody asks you a question in an interview like this, you should begin at the most elementary level. So I started with the coin-tossing space, which I really like. Then he asked, "Do you consider the coin-tossing space to be the most important stochastic process?" I then had to talk about Brownian motion. After that, he asked something about Donsker's theorem that I still remember. Some questions were also from linear algebra.

More or less, that's it. That was my interview.

Is there any anecdote from the time when you were deciding between TIFR and ISI Delhi?

PSC: I will tell you one more story about my memorable trip from TIFR to ISI Delhi. I remember it quite vividly. One of my teachers, who is now the Director of ISI, was very excited and wanted me to join TIFR. He asked me, "Have you seen the Western Ghats?" I said, "Yes, I have seen the Western Ghats, but what is so great about them?" I did not quite realize his point at the time.

Southwest Monsoon at TIFR. Anveshanā

That night, on my way from Bombay to Delhi for my interview, Bombay experienced one of its torrential rains, and all the trains were either cancelled or delayed. At the station, they did not announce that my train had been cancelled. It was scheduled to leave at 5:00 p.m., and even at 7:00 p.m. they were still announcing that it was on time. I learned a funny rule: they could not utter a word without approval from the higher authorities.

Anyway, I stood on the platform throughout the night. Finally, around five in the morning, the train arrived. Everybody rushed in and quickly fell asleep. When I woke up five or six hours later, the train was standing at a station. To my surprise, it was still at Victoria Terminus! Eventually, the train started. Because of the rain, the entire Western Ghats looked serene. There were countless small streams flowing down the hills.

It was a very memorable journey. Looking back, I realize it was a wonderful experience. After all, you cannot really arrange a sight like that, with such heavy rain.

And do you remember the last time you met K. R. Parthasarathy?

PSC: I last met him, probably in 2016. That must be my last visit to ISI Delhi. After 2018, I have never really left Kolkata, even for a single night.

"I learned a funny rule: they could not utter a word without approval from the higher authorities."

Where have you been in your education, starting from undergraduate?

PSC: Just after my schooling, I entered ISI Kolkata. I did my B. Stat and M.Stat—everything here. Then, for my Ph.D., I joined ISI Delhi.

Before joining ISI Kolkata, did you always see yourself becoming a mathematician?

PSC: Yes, but that was not the plan. As a kid, I always wanted to study physics. In the VII or VIII standard, in physics books, you see the names of physicists, for example, Jagadish Bose and Meghnad Saha, and then you begin to learn about their lives. You generally do not really learn about their physics at that age, but you certainly learn about their lives, and there is a certain amount of romanticization. This made me want to pursue physics. But during higher secondary, I realized that physics practicals were quite difficult for me to perform. On the other hand, I also realized that mechanics problems attracted me more.

I always thought that I would study physics at Presidency College. And there was no doubt about it. Since everyone was writing joint entrance tests, I also thought I should write; otherwise, people would say I could not clear the test, and that is why I joined physics. I wrote joint entrance exams, including the medical school exam.

But one of my teachers happened to tell me, "If you like mathematics so much, why don't you write the ISI entrance exam?"

ISI Kolkata. Anveshanā

The most exciting thing about the ISI entrance exam is that it tests only mathematics, not any other subjects. That was very interesting, but another interesting point was receiving a stipend while studying. I would not have joined ISI if there had been no stipend. Maybe I would have gone straight to the Presidency.

So that was when you decided to join the B.Stat program at ISI?

PSC: At that time, that was the only possibility. It was the best mathematics program in India. Moreover, TIFR was only hosting Ph.D. programs. While Integrated Ph.Ds were also available, very few people chose them, as they were not very popular back then. Indranil Biswas was one of our illustrious seniors. Even he did his M.Stat and then joined TIFR.

Let's rewind a little. How did your interest in science and mathematics first develop?

PSC: I think my father told me the following. In his time, the belief was that good students did not pursue engineering or medicine; instead, they studied in the general sciences and then went on to research. So, it was more or less decided. Besides, I do not think I was very good at languages such as Bengali and English. I thought that in science, one writes precise answers, not really essays, so this was probably why the sciences attracted me.

I liked mathematics from a very early stage. I remember some relatives were visiting us, and as people often do, one of them asked what my favourite subject was. My mother responded that she thought it was mathematics. I still remember that, from then on, I used to think, "If my mother thinks that mathematics is my favourite subject, then it must be so." I used to do all the problems from my elder sister's books, so it was quite clear that I would pursue mathematics. At that time, most of the good students used to choose science. I now think good students should study philosophy, or at least reflect on philosophical questions. Of course, the sciences are somewhat important; one should really like to know how things work. It is a fundamental question. Physical realities surround us, and one cannot escape the attraction of questions about the natural laws governing their behaviour. Yet mathematics feels even more fundamental.

Mathematics allows you to deduce things without doing any experiments—just pure thought leads you somewhere. I mean, pure thought is really good enough for many purposes. When I first started reading books on probability, I imagined predicting the weather, and that would certainly be magical, right? At the time, that was what I thought probability was about. But once I entered ISI, I started doing very abstract mathematics and moved away from that connection with physical reality.

When you start doing mathematics in a very serious way, there is a risk of losing contact with real-life phenomena. It is probably a good idea to keep that connection intact, as it makes the subject more charming. For a while, I, too, felt that pure mathematics has its own attraction, dictated by logic. Through your attraction to logical ways of doing things, you become removed from natural or physical realities. But later on, you realize that perhaps one must reconnect with physical realities. Otherwise, it becomes a purely idealized setting. This is what I feel now.

"Mathematics allows you to deduce things without doing any experiments—just pure thought leads you somewhere."

For example, while I was teaching complex analysis last semester, I thought I had to bring that connection into the course. The final problem I gave was on Kepler's equation. I told the students that we are developing these concepts in complex analysis, which can be used to solve useful equations.

So, initially, I was attracted to physics but ended up immersed in mathematics. Eventually, I moved back towards physical realities. I think this second shift was influenced by people like KRP and my supervisor, Kalyan Bidhan Sinha [KB Sinha].

How did you eventually decide where to pursue your Ph.D., and what led you to choose ISI Delhi?

PSC: It was a very simple decision. After my master's degree, I decided to pursue a Ph.D. At that time, I was essentially weighing two possibilities.

I did write the ISI entrance test, but I was not sure whether I would join. On the one hand, I was considering TIFR. It was regarded as the best place in India at the time. But there was another possibility that I was seriously considering—working in stochastic finance.

After my master's, I met my one-year senior, Debashish Goswami, who was still at ISI Kolkata and had just begun working with B.V. Rao. During our conversation, he mentioned that he was planning to change his supervisor, move to Delhi, and work with KB Sinha. He also spoke about the mathematical direction he intended to pursue, referring to the Atiyah–Singer Index Theorem and Bismut's probabilistic proof, and suggested that I consider joining him in that direction.

Kalyan Bidhan Sinha AMBARNSG THROUGH WIKIMEDIA COMMONS

He then mentioned that noncommutative geometry has index theorems, and that ISI Delhi had experts in quantum probability. The idea was to prove some of these noncommutative geometric index theorems using quantum probability.

He already knew that I was very interested in differential geometry. I had been studying it on my own since my second year of the B.Stat programme. He thought that I might be able to contribute to that programme. It looked like a golden opportunity. So I immediately decided to join ISI Delhi.

Was that your first encounter with noncommutative geometry?

PSC: Yes. Before deciding to join ISI Delhi, I had never even heard of noncommutative geometry.

Because stochastic processes were very important at ISI Kolkata, all the time you heard about stochastic processes, Brownian motion, and stochastic integrals. People almost thought that was the entire world. Similarly, at that time, we used to think that this was the ultimate goal one could achieve.

What was it like to be a Ph.D. student ISI Delhi with KB Sinha and KR Parthasarathy?

From left: Debashish Goswami, Surya Jana, and Partha Sarathi Chakraborty PARTHA SARATHI CHAKRABORTY

PSC: Those years were some of the most memorable periods of my life. The environment at ISI Delhi was very friendly. There were very few Ph.D. students at ISI. In the year before me, only Debashish [Goswami] joined. In my year, only I joined. I believe that since there were so few of us, the teachers really celebrated the Ph.D. students. We would spend time together throughout the day. Every morning, I went to my shared office. It was all on the ground floor of the ISI Delhi building. It was I, Debashish, KBS, KRP, and Arup-da [Arup Pal]—who was a postdoc there at the time, and he joined as a faculty member in June 1998. We would really spend the entire day together, right up until 5-6 in the evening. Our offices were right next to each other.

KRP would come to the Institute around 10 o'clock. He was very punctual about that, as I have already shared about his love for routine. And now and then, he would come into our office, go to the board, and state a problem. These sessions would happen between 10 and 11. Then, around 11 o'clock, all of us would go for tea together and discuss some mathematics. One day, KRP would state a problem, the next morning, he would solve it, and then state another. It was very difficult to keep pace with him. Also, at that time, our interests were different. He was interested in quantum information theory, probably because his doctoral thesis was on information theory.

> *"One day, KRP would state a problem, the next morning, he would solve it, and then state another."*

After working on other topics, like quantum probability, around 1997 or earlier, just after Raja-da's thesis, I think he became interested in quantum information theory and cryptography. He became very interested in topics such as encryption and was constantly posing problems in that area.

On the other hand, at that age, you think that whatever you are doing is the greatest mathematics. Of course, we were interested in noncommutative geometry and were trying to read more about operator algebras. So, we could not really spend much time on his problems.

Anyway, we would come back to the office, and then all of us would go for lunch together at one o'clock. Debashish and I were talking throughout the day. In the hostel, two rooms shared a common bathroom. So, we were also bathroom mates.

Looking back, how do you feel about your decision to stay in India for your Ph.D.?

PSC: Not really. At that time, I was very confused about whether to go abroad, but I liked my mother's argument. She said, "Anyway, it is something you have to do. You have to write your thesis. Therefore, how should it matter so much whether you do it from India or you go abroad?"

I found this argument convincing. And as Amartya Da [Dutta] told me, if you go abroad, you have to manage your life–you have to cook your own food, and these things could be difficult. In India, you have a hostel where you get cooked food, and everything is taken care of. But in some sense, you get better exposure if you go abroad. However, there are advantages to being here as well.

> *"Every situation turns you into a unique individual."*

Meaning, a man is really forced by the situation he faces. Every situation turns you into a unique individual. So the situation could have been different if I had gone abroad. I might have pursued something very different. Stochastic finance is far removed from what I do. But on the other hand, the subjects that I pursued, I do consider magnificent. It connects several ideas from vastly different sources.

The idea of non-commutative geometry is an amazing one, and it is really by no means an obvious development. It is a highly sophisticated development. The theory is rooted in index theorems, so having a full appreciation of it is quite difficult, and I really enjoyed figuring it out.

Having spent time at both ISI Kolkata and ISI Delhi, what differences did you notice in the way mathematics was approached at the two institutes?

PSC: Actually, at institutes like ISI Kolkata and ISI Delhi, people think differently. I will give you a very simple example.

I did my master's at ISI Kolkata, and I remember a specific situation involving the two of us. Debashish did his second year at ISI Delhi and had his Ph.D. interview in Kolkata. I studied functional analysis during my master's at ISI Kolkata, and I had my interview at ISI Delhi. We both faced the same question in our interviews.

Partha Sarathi Chakraborty, Surya Jana, K.V.S. Vinay, and Sourav Mukhopadhyay (M.Stat Students) in front of the statue of PC Mahalanobis. Partha Sarathi Chakraborty

Debashish was asked what he knew about compact operators. He started answering by describing compact operators on a Hilbert space. And when I was asked the same question, I started answering by describing compact operators on a Banach space. This happened because at ISI Kolkata, under the influence of Professor A.K. Roy, the emphasis was heavily on Banach spaces. There, you start with Hilbert spaces, and then you prove that a Hilbert space has a unique invariant, namely the cardinality of its orthonormal basis. I don't think it was explicit, but the implicit message I got was that the theory of Hilbert spaces is over. Let us move to Banach spaces.

On the other hand, when you go to ISI Delhi, they are interested in operators on Hilbert spaces, and they say that a Hilbert space has a life of its own. Any two infinite-dimensional separable Hilbert spaces are unitarily equivalent, but they are not quite the same. For example, $L^2(\mathbb{R})$ and $L^2(\mathbb{Z})$ are not really the same thing.

That was a major change in our attitude. At ISI Kolkata, the emphasis was on what I would call pure mathematics. The moment one went to ISI Delhi, one became a member of what KRP used to call the quantum group. So, there was a clear-cut transition in how we were doing mathematics.

But what do you mean by transition in the style of doing mathematics?

PSC: Now again there is another question—what did we mean by doing mathematics? At ISI Kolkata, people were concerned about very abstract mathematical questions. For example, consider the Borel isomorphism theorem. You take two sets and ask whether they are Borel isomorphic, and you learn about such things. These questions felt completely removed from reality, or at least that was how it appeared to me at the time. One should not take that past judgment too seriously now, but back then it was posed to us as a purely mathematical question, with no connection to physical realities.

On the other hand, the moment you went to ISI Delhi and talked to people like KRP or KBS, they immediately started connecting things.

Do you recall such an interaction?

PSC: I remember a shocking discussion with my supervisor. Debashish and I were both sitting in front of him, and he [KBS] asked us, "Do you know Newton's laws?" We said yes. Then he replied, "Do you know Newton's third law?" We said yes, of course. He asked us to give some examples. Naturally, we told him about how we were standing on the ground and so forth. Then he asked us to imagine the following scenario. Think of the frame of a table, but instead of a solid wooden board on top, we put a piece of paper and ask you to stand on it. Now, "If Newton's third law is valid, then what will actually happen? Of course, the paper will fall apart, right? It will tear, and you will fall through."

With KB Sinha in front of a cafe. Partha Sarathi Chakraborty

He explained that the first and second laws are about force. They provide a qualitative definition of force, followed by a quantitative expression. But the third law is, in fact, the qualitative definition of a rigid body. But of course, rigid-body motion is much more difficult to calculate. And this is what our interactions with them were like.

"It was a massive transition in how I viewed mathematics."

Since they understood physics so deeply, they would connect abstract mathematics to real physical systems from time to time. That level of mentorship led to a major change in my attitude. It was a massive transition in how I viewed mathematics.

How close were they to mainstream physics?

PSC: I am not quite so sure about that. Of course, they were interacting with many mathematical physicists, but you realize that mathematical physics is not quite physics, right?

But I am not sure about their interactions with experimental physicists, but I am sure they were in touch with theoretical physicists, particularly those from Europe.

Did ISI have active theoretical physicists at that time?

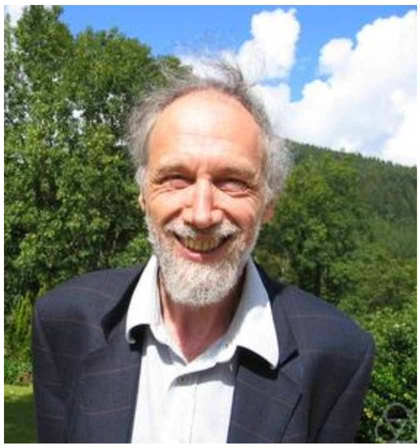
PSC: No. ISI does have a physics department, but I do not know about theoretical physicists. I did take electives in physics, but it was not really that appealing.

ISI Delhi did not have a physics department. ISI Kolkata did have one, but I think the main difficulty was language. See, the mathematicians are too keen on such things. For example, when you meet my supervisor, he will say that self-adjointness is a big issue. If you are a mathematician, indeed, self-adjointness is a big issue. Symmetric operators and self-adjoint operators are not quite the same thing. And, physicists do not make those distinctions all the time. Physicists are not really bothered about these mathematical issues all the time. Of course, they had some connection with physics, but at the end of the day, both KRP and KBS are really mathematicians.

What was your first exposure to non-commutative geometry like, and which aspects of the subject impressed you most initially?

PSC: At the time I started reading the subject, there were not many books. As I joined ISI, the first paper I was asked to read was [Alain] Connes' IHES paper. That, I think, must be his first paper on the topic. I believe so. It could be a controversial statement, but I believe it must be so, because that's where he is talking about the program, right?

The paper[3] is titled "*Non-Commutative Differential Geometry*." There, he discusses the program and introduces fundamental concepts such as cyclic cohomology. He demonstrates his new subject by



Alain Connes at Oberwolfach in 2004.
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[3]Connes, Alain. "Non-commutative differential geometry." Publications Mathematiques de l'IHES 62 (1985): 41-144.

giving a beautiful solution to the no idempotent conjecture for free groups on two generators. The conjecture was the following. You consider a free group on two generators, and then in its group ring, he proves that there are no non-trivial idempotents. As you state this problem, it looks purely group-theoretic, because you have a group that fits into a general conjecture. The conjecture is the following. If a group has no torsion elements, then its group ring has no non-trivial idempotents. And apparently, as you look at this problem, you do not see any connection to any geometric ideas. But it is amazing how he makes all those connections and proves it.

And even yesterday, I was trying to convince students about the greatness of the subject by telling them about his [Connes'] argument. Of course, this conjecture was open for about 20 years, and it was settled by Pimsner and Voiculescu. Connes also gave his absolutely cute argument. And then, in that article, he talked about his program, which aimed to answer some very interesting questions. And so the subject looked really very inspiring, I would say.

Would you describe non-commutative geometry as a generalization of classical geometry, or as a fundamentally different viewpoint?

PSC: It is a very different way of looking at things, and is not at all a trivial generalization. The idea that you can capture spaces by cycles in some suitable homology theory is not very uncommon. In algebraic geometry, they study cycles. But the fact that you can encode very fine differential geometric ideas through some very specific cycles is not at all obvious.

"It is a very different way of looking at things, and is not at all a trivial generalization."

Connes, I think, wrote his book around 92-94. There, he mentioned a reconstruction theorem for classical spaces. The proof to him was clear, as many things could be, but he did not really bother to write them down. At the same time, other people were trying to prove it.

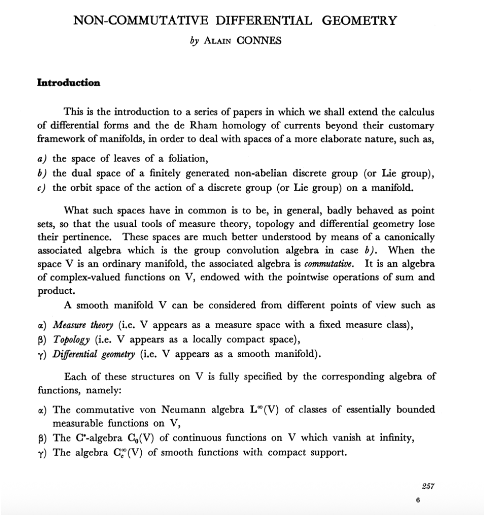
Connes' remarks must still be publicly available, noting that someone has attempted it, that there are mistakes to be corrected, and so on. So eventually he wrote down his proof. His proof of the reconstruction theorem was published around 2012. It is a highly non-trivial idea that you can encode all the information of a Riemannian manifold, or a compact Riemannian spin manifold, into some sort of functional analytic data.

The framework of noncommutative geometry brings together analytical, algebraic, and geometric structures.

PSC: : Yes, all these ideas are kind of coupled in a rather fine way; to a certain extent, it is a mixture of analytical and algebraic ideas. Because you have a self-adjoint Dirac-type operator with compact resolvent, you have an algebra. Now, you start making very strong demands on them, and they are also algebraic in a certain sense, and they produce certain modules which are subsets of a Hilbert space, which are finitely generated, projective, etc.

How should one understand the relationship between the C*-algebraic formulation and the spectral triple framework in non-commutative geometry?

PSC: I do not think that there is any difference between the two approaches. A commutative unital C-star algebra is really a space of functions on a compact Hausdorff space. If you remove the commutativity, you have the analogs of non-commutative spaces.



NON-COMMUTATIVE DIFFERENTIAL GEOMETRY

by ALAIN CONNES

Introduction

This is the introduction to a series of papers in which we shall extend the calculus of differential forms and the de Rham homology of currents beyond their customary framework of manifolds, in order to deal with spaces of a more elaborate nature, such as,

a) the space of leaves of a foliation,
b) the dual space of a finitely generated non-abelian discrete group (or Lie group),
c) the orbit space of the action of a discrete group (or Lie group) on a manifold.

What such spaces have in common is to be, in general, badly behaved as point sets, so that the usual tools of measure theory, topology and differential geometry lose their pertinence. These spaces are much better understood by means of a canonically associated algebra which is the group convolution algebra in case *b)*. When the space V is an ordinary manifold, the associated algebra is *commutative*. It is an algebra of complex-valued functions on V, endowed with the pointwise operations of sum and product.

A smooth manifold V can be considered from different points of view such as

α) *Measure theory* (i.e. V appears as a measure space with a fixed measure class),
β) *Topology* (i.e. V appears as a locally compact space),
γ) *Differential geometry* (i.e. V appears as a smooth manifold).

Each of these structures on V is fully specified by the corresponding algebra of functions, namely:

α) The commutative von Neumann algebra $L^\infty(V)$ of classes of essentially bounded measurable functions on V,
β) The C^*-algebra $C_0(V)$ of continuous functions on V which vanish at infinity,
γ) The algebra $C_c^\infty(V)$ of smooth functions with compact support.

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Facsimile of the first page of Connes' famous 1985 IHÉS paper IHÉS

But what does it mean to say "doing geometry"? Look at what algebraic geometers do — they study cycles. That means, doing geometry should amount to studying some cycles. That idea was known. Even Atiyah had posed this problem; he called these things "abstract elliptic operators," but he did not quite describe the equivalence relation that turned it into a homology theory. Kasparov did that. Connes then realized that considering unbounded cycles could capture even finer information.

Another key point is the pairing between cycles and K-theory. It was already known since the 70s that K-theory could be done in the framework of Banach algebras. Karoubi wrote his book, K-Theory for Banach Algebras, in the 70s. The pairing between K-homology and K-theory was also known from Kasparov's work. Connes's remarkable construction was cyclic cohomology, in which he introduced the Chern character and the pairing, and this was the connection to the spectral triple. The spectral triple is really an element in K-homology, but with finer properties. Not any cycle will do, and not all cycles carry the same amount of information. There are certain good cycles, and then using them, you can construct certain meaningful things.

Then you ask: can you really encode all the information in a few very good cycles? That was his [Connes] reconstruction theorem. Spectral triples are therefore not far removed from C*-algebras. If you have a C*-algebra, you have its K-theory and K-homology groups, and elements of K-homology groups are the spectral triples. Certain spectral triples contain more information, and those are the candidates for non-commutative geometric spaces.

Non-commutative geometry was also motivated by ideas from quantum mechanics and particle physics.

PSC: Connes did something remarkable about which he was very excited. He derived the standard model from his Yang-Mills action and presented a rather conceptual derivation as an optimization principle, or a least-action principle.

It has also influenced areas such as string theory and condensed matter physics.

PSC: It has its reach among physicists. Maybe not that much right now. But it is a convincing argument, more or less, to many people.

Have you met Connes?

PSC: Yes, I have when I was visiting IHES for longer durations.

"But what does it mean to say 'doing geometry'?"

During your Ph.D., your work focused on quantum probability and aspects of noncommutative geometry. Could you describe the main directions you worked on at that time?

PSC We started with that, and my first paper was on the topic. I think it was published in the Journal of Operator Theory.[4] In quantum probability, one starts with a semigroup of completely positive maps and dilates it into a flow of homomorphisms. We constructed a CP semigroup on the noncommutative torus and dilated it. We also studied perturbations of spectral triples, though we were not yet very familiar with the homological background. We proved that the perturbation is not unitarily equivalent to the original, though later one realizes that unitary equivalence is not the right equivalence relation.

After some work on an equivariant version of a problem in quantum probability, I did not pursue the subject much further because its framework did not seem compatible with noncommutative geometry. I could not see any connection with these cycles. Although Jean-Luc Sauvageot and Cipriani later produced something, the two areas largely lived their own lives. There was nothing like Bismut's work connecting them.

What did you explore mostly?

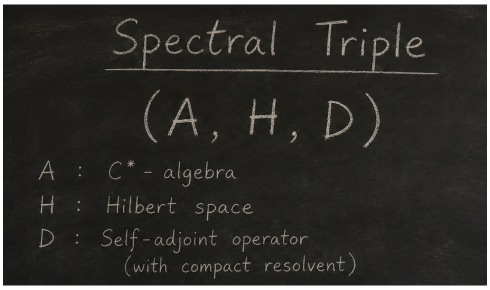
PSC: So I explored two questions: the quantum Heisenberg manifold, which I liked a lot, and embedding quantum groups into noncommutative geometry. At the time, the prevailing belief was that quantum groups were not compatible with the framework, and there was correspondence suggesting that it might need to be modified. Arup Pal—whom I refer to as Arup Da—and I disagreed. There was no proof of incompatibility; only some obvious guesses had failed. We thought that if we tried hard enough, we might prove something. And we did—we proved that quantum SU(2) admits spectral triples.

"We thought that if we tried hard enough, we might prove something."

Of course, there are further technicalities. One asks whether a spectral triple is finitely summable. This is an analytic property that abstract homology does not provide. We

[4] Chakraborty, P. S., Goswami, D., Sinha, K. B. (2003). Probability and geometry on some noncommutative manifolds. Journal of Operator Theory, 185–201.

constructed a finitely summable spectral triple representing a non-trivial K-homology class. We submitted the paper to K-Theory[5], and I believe Alain Connes was the referee. Our argument was rather intricate. Connes observed that the required properties of the operator could be verified directly and confirmed that it was indeed a non-trivial spectral triple.



A set of data is a *spectral triple* with some *properties* satisfied. ANVESHANĀ USING CHATGPT

At the time, he was interested in illustrating the local index formula he had proved with Moscovici in 1997. To apply it, one needs spectral triples satisfying regularity and possessing a finite-dimensional dimension spectrum. Connes proved that our spectral triple had these properties, computed its local index formula, and made some remarkable discoveries. For instance, the Dedekind eta function appears as a coboundary given by the difference between two cocycles representing the same class. Why this happens is still not entirely clear, and probably a more general framework is needed to explain it.

After that work, it became clear that quantum groups can produce finitely summable spectral triples, particularly under equivariance. Neshveyev and Tuset later constructed such spectral triples for $SU_q(N)$ and general compact quantum groups, but I do not think their regularity has been verified. Computing the dimension spectrum and the local index formula in that setting remains open. That work may eventually explain the appearance of the Dedekind eta function in this computation.

How should one evaluate a large mathematical program, such as non-commutative geometry or the Langlands program? What, in your view, constitutes success in such a program?

> *"Our task, then, is to recognize structures and to develop the language to describe them."*

PSC: I think that new definitions and examples are important, especially today. It makes me wonder: what is the purpose of a mathematician in the near future? As kids, we enjoyed solving geometry problems by looking at a picture and connecting the dots in our minds. Now, there may be assistants capable of making such deductions. If so, perhaps our role lies elsewhere.

Our task, then, is to recognize structures and to develop the language to describe them. There are many highly non-trivial structures, both abstract and arising from the real world, that

[5] Pal A, Chakraborty P. Equivariant Spectral Triples on the Quantum SU(2) Group *K-Theory*. 2003;28(2):107-126.

remain to be understood. The nature of mathematics will become much more complex, and perhaps we are now in a position to address it. Earlier, for instance, children spent time memorizing multiplication tables. I am not sure whether children today still memorize them. If they still do, I do not quite see why, given that we now have very effective tools for such tasks.

In doing mathematics, we will likely have very powerful machines at our disposal. What we will need, therefore, is the right language. Languages emerge from examples and develop to describe structures. Formulating the appropriate language is not easy, and it may well be the central challenge. At present, we prove propositions and develop our own understanding of structures. In the future, we will likely have powerful assistance in doing so. However, I do not think machines will be able to formulate the language itself. They may be effective at problem-solving, but building a theory is a different task. It is not entirely a question of theory-building versus problem-solving, though there is some overlap. Theory-building is about finding the right language, and its formulation will remain essential. I remember

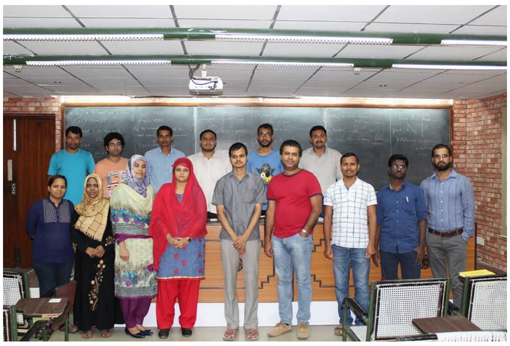
Vaughan Jones saying, "Location, location, location, that's the key consideration, as far as property is concerned. Location is the first word and the last word." He would then say, in mathematics, "Notation, notation, notation," and that "notation is the first word and the last word." That conveys how fundamental language and notation are in mathematics. He was speaking about planar algebras, where the notation is indeed remarkable.

Oh, have you met Jones?

PSC: : Yes, I met him.

He was a wonderful barista as well.

PSC: Indeed, he came to IMSc—more than once. He also visited when I was a faculty member there.



At a summer school at IMSc organized by Partha Sarathi Chakraborty. PARTHA SARATHI CHAKRABORTY

On that, you joined IMSc. How did that transition happen, and what drew you to Chennai?

PSC: IMSc was very nice. I had never imagined that I would go to Chennai for a job; it was not something I had really considered.

When you grow up in India, you tend to think of TIFR as the ultimate destination. So, when I was applying for jobs, I applied to TIFR and to ISI. Then I met V. S. Sunder, who asked me, "Why did you not apply to IMSc as well?"

Then I applied to IMSc, and they made a very attractive offer. I decided to join, and it turned out to be an excellent decision. One of its strengths is that it is an extremely friendly and relatively small institute. Everyone knows everyone else, and there is a very friendly atmosphere throughout the institute.

Was this your first employment after your PhD?

PSC: Well, it was my first permanent position. After completing my PhD, I went to the UK for a postdoctoral fellowship funded by the Royal Commission for the Exhibition of 1851 at the University of Cardiff. I was originally supposed to go to Paris for my first postdoc. However, some complications arose regarding the transfer of funds, including a bureaucratic delay. From Cardiff, I moved to IHÉS for an extended visit. I completed my postdoc in 2005 and then came to ISI Delhi for a short visit of about six months. After that, I joined IMSc.

In front of a train. Partha Sarathi Chakraborty

IMSc also has a very lively physics department.

PSC: In fact, IMSc had some very interesting traditions. One of them was called the Institute Seminar Week. Every year, people from different disciplines give a 20-minute talk. In twenty minutes, of course, you cannot say very much. The idea was not to go into technical details, but to tell colleagues from other areas what you were working on and why it was interesting. I thought that was a wonderful idea.

At ISI, I have often told my colleagues that we should have something similar. Unfortunately, I have not been able to make it happen. Perhaps I have not done a good enough job of convincing them of its value.

I think such initiatives help people understand the basic ideas and problems that their friends and colleagues are thinking about. We should make an effort to communicate. After all, if we cannot communicate effectively with fellow scientists, how can we expect to communicate with the broader public, with bureaucrats, or with anyone else? We should be able to explain what we are thinking about, why it matters, and its significance.

> *"...if we cannot communicate effectively with fellow scientists, how can we expect to communicate with the broader public, with bureaucrats, or with anyone else?""*

What was your experience at IHÉS like? Did you interact with Konstevich and other people?

PSC: Yes, of course. IHÉS is a truly wonderful place. Anyone who spends time there tends to enjoy the experience immensely. My first visit lasted six months; I returned later on two more occasions. It is an environment where everything is conducive to good work–you have excellent food, excellent company, and a vibrant intellectual atmosphere. What more could one ask for?

What led you to decide to move back to ISI Kolkata after your time at IMSc?

PSC: In 2018, after spending twelve years at IMSc, I moved to ISI Kolkata. I had a wonderful experience at IMSc. I thoroughly enjoyed my time at IMSc, but as my parents were getting older. I decided to move to ISI Kolkata.

Do you think there is increasing fragmentation within mathematics, even between different subfields?

PSC: Yes, there is. This is not a particularly new idea. You may recall Gian-Carlo Rota's article[6], "Ten Lessons I Wish I Had Been Taught." One of his suggestions was that a mathematician should write the same paper several times, each version aimed at a different audience and level of accessibility. I think there is a great deal of wisdom in that advice.

I do believe that communicating ideas is an important responsibility of scientists and mathematicians — whether to students, to the general public, or to colleagues in other disciplines.

For example, consider a recent case where a computer program solved a longstanding problem in geometry involving distances. What was striking was not only the solution itself, but also the way it drew upon ideas from several different areas of mathematics. It concerned the Erdős distinct distances problem and involved techniques connected to algebraic number theory. Developments like these show how deeply interconnected mathematical fields can be. Precisely because of these connections, we must explain our work beyond our immediate area of specialization. Otherwise, many interesting ideas and insights remain inaccessible even to other scientists.

> *"What seems obvious or elementary to you may be quite non-trivial and illuminating to someone else."*

When someone is invited to give a talk, they often feel they should present the most sophisticated result in their area. The result may be impressive, but conveying its significance to people outside the field can be difficult.

Sometimes it may make more sense to discuss something that appears trivial to you. What seems obvious or elementary to you may be quite non-trivial and illuminating to someone else. The same applies in the other direction as well.

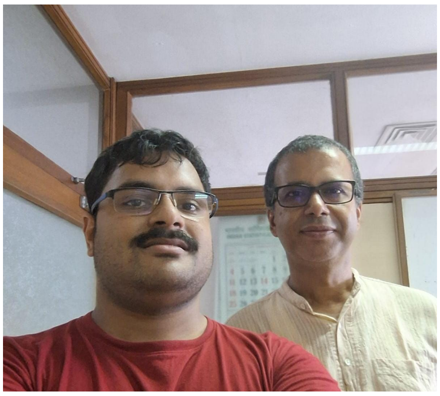
In a sense, this is part of what it means to do mathematics today. Mathematics is not only about discovering new results; it is also about communicating ideas, building connections,

[6]Available at https://www.ams.org/notices/199701/comm-rota.pdf

and helping others understand why certain questions matter. I think that is certainly one of the responsibilities we should take seriously.

What does "mathematical maturity" mean to you, especially in the context of training future researchers?

PSC: Well, this has to be judged differently in different cases. I cannot give you a single formula for assessing mathematical maturity, because we do this all the time, and it depends heavily on the individual student and their background. In fact, I would distinguish mathematical maturity from aptitude. When someone solves a combinatorics problem quickly, for example, that may reflect aptitude, but not necessarily mathematical maturity. Maturity is more closely related to one's background, preparation, and understanding of mathematics.



Partha Sarathi Chakraborty with his student Saraswata Sensarma ANVESHANĀ

So, when we talk about mathematical maturity, we have to take into account the person's prior training. Given a student's background, one develops certain expectations about what they should know and how they should think about mathematics. After an undergraduate program, one would expect a reasonable degree of familiarity with a few fundamental subjects. In particular, I mention three broad areas: linear algebra, calculus, and some basic algebra. That would be a natural expectation.

How should a young mathematician approach the choice of research problems that deserve a lifetime of thought?

PSC: On this question, different people have different thoughts. I think TIFR popularizes this idea. There, as one joins a PhD program, one goes through very serious training for one year. And probably they do that because they expect every mature mathematician to know certain results. So, that is not a bad idea. What that does is allow more serious communication across diverse areas of mathematics.

"But sooner or later, one has to choose a problem wisely. This is a difficult decision."

A mathematical researcher should have a good idea of a reasonable introduction to algebraic topology, a good introduction to measure theory and functional analysis, and some reasonable algebra and commutative algebra. Non-commutative algebra is not that common, but differential geometry, some Lie groups, and Lie algebras, for example, are things everybody should know to a certain degree.

To become a mathematical researcher, you should have a basic understanding of many of these topics. But sooner or later, one has to choose a problem wisely. This is a difficult decision. Why?

Because, from my experience, more often than not, choosing a problem is really an emotional decision. For most people, it is. Atiyah has done that, and he is a Fields medalist. To understand the problem, one already has to invest a good amount of time in that area. If one is interested in problems on the frontier, one really has to dig deeper into an area. So one has to invest a year or so, or more. And then it is not easy to change from one frontier to another. Therefore, one already has to make some sort of emotional decision. Of course, nowadays there are various other ways to decide.

I no longer suggest that my students choose problems the way I did; I simply rule that out. When we were students, even reading a mathematical review was difficult because we had them in hard copy. Mathematical Reviews (AMS) used to be some big, fat, orange-colored books where one physically browsed through reviews. These days, with a click of a button, one can see who has referred to a paper, what kind of work is happening, in which journals it is appearing, and so on. A much more mature decision is possible now. It was not possible when we were students, but now one can check what kind of work is happening in these areas.

With Rishiraj Baul and Satwata Hans. Partha Sarathi Chakraborty

I am suggesting that probability is the key area in which it makes sense to invest. And I am telling all my students, invest in probability. But then, I have to justify it to them. If you check the schedule of talk[7] at ICM, you can figure out why one should invest in probability, and now you can make a mature decision.

"A much more mature decision is possible now."

These days, we have access to information, and analyzing it is much easier. Therefore, one can make a mature decision today in that sense. And probably one should do that. But there are risks too. For example, nobody will think about doing something that is fundamentally

[7]There are 10 talks in '12-Probability' section of ICM list at https://www.icm2026.org/event/ac193975-5d24-4628-8c30-ddb23de19a8b/speakers.

different from what is happening and is rather far removed from the center of activities. Those possibilities are somewhat reduced. But I'm sure there will be people sitting in some corner trying to do all sorts of things. That possibility is now with established researchers. But these days, even aging people come up with interesting results. But people still think that mathematics is a game for young minds. For being a very serious mathematician, one has to really learn a lot, and therefore should not take that comment very seriously.

Has your approach to suggesting problems to students evolved over time, or have your expectations about when a student is ready to tackle a problem remained the same?

PSC: Not very much. There are certain minimal requirements: the student must be familiar with certain topics, and then they can tackle a problem. So I do not think that view has changed. The requirements I would have considered sufficient for tackling a problem then are more or less sufficient even today.

Do you still engage with music, or has it remained mainly a part of your earlier years?

PSC: I learned how to play sitar for a while. Around the 11th or 12th standard, I realised the sitar is a very big instrument[8], and you cannot really move around with it. Therefore, that was not a good idea, and I ended up discontinuing. During my PhD, I had some time, so I started learning the flute. A flute is an easy-to-carry instrument, and it may not be as magnificent as a sitar, but it is still an interesting instrument. So in my free time, I enjoy playing the bamboo flute.

As a child playing Sitar at the annual program in school. PARTHA SARATHI CHAKRABORTY

Do you read? If yes, how do you find which books to read or not?

PSC: I enjoy reading. These days, books are also very easily available, and once you start reading, agents are exploring your mind, and you should let them explore. I should not resist. You should let them explore, and these agents, at times, I would say, make good suggestions. For example, Facebook does that. For instance, it suggested that Princeton University Press has a Facebook page. You can check that page and learn about their latest publications, and they promote books that are of interest to you. So that means some machine in some corner has figured out your mind. That is perfectly all right,

What kind of literature and books do you read?

PSC: There are various kinds of books, but there are certain philosophers whom I cannot deny. I think there is an underlying philosophy in our reactions. We may not make it explicit, and we may not even realize it, but every human life reflects philosophical thoughts about life itself.

[8]The average Ravi Shankar-style sitar measures about 48 to 51 inches in length and weighs roughly 3 to 4 kg.

In the academic profession, we have certain luxuries: some free time to think. At such times, one learns what others are thinking. Given any issue, one can ask philosophical questions — about the nature of life, the meaning of human life. These are philosophical questions, though we now approach them with very different methods. Large language models, for example, allow one to ask whether life has any meaning.

For example, consider the DNA sequences of living creatures. Perhaps these are all expressions in some language. Is this language evolving in some sense? How is it evolving? These questions have become relevant. Can the large language models be employed to understand the evolution of these DNA sequences? Will that shed new light on the meaning of life itself?

"The ultimate realization is not expressible in language."

This leads to Wittgenstein's question: are there things beyond language? The last proposition of the Tractatus says, "Whereof one cannot speak, thereof one must be silent." Here precisely, he is hinting about things beyond the reach of languages. We are now seriously exploring language, perhaps for the first time at this scale. What does it mean for something to lie beyond language? Are there things not expressible through language?

This resonates with certain spiritual ideas. For example, one cannot speak of Brahman in language; it is an experience. Ramakrishna Paramahamsa gives the example of sugar: whatever you say about it, you cannot explain it unless one tastes it. The ultimate realization is not expressible in language.

Living a life in math, what do you think a mathematician can learn from people who devote themselves to other vocations other than math and academia in general?

PSC: Let us begin by asking: what did von Neumann think when [Oskar] Morgenstern approached him? He thought about game theory and human behavior. When a mathematician thought about human behavior, he formulated game theory—a mathematical way of approaching it. This is a fundamental lesson: whenever we observe something happening elsewhere, a mathematician translates it into a precise language. This connects with the earlier point about devising the correct language.

Mathematicians should interact with people from other areas and assist in formulating the appropriate language. In this way, one obtains many new problems. Once problems are formulated mathematically, their analysis can be assisted by machines, enabling deeper mathematical exploration of the world. So far, many of the problems we have tackled involve simplifying assumptions and removing ourselves from real life. We can now address more complex structures through an appropriate mathematical language.

If a machine can speak as I do, then the language I am using is not very complex. Then one can move towards more complex problems by translating them into a precise linguistic framework. At some point, such languages may go beyond our unaided capabilities. However, with the assistance of machines, we should still be able to understand them to some extent.

An example of a complex structure is soil. Gromov, in one of his lectures, remarked that we do not know how to treat soil mathematically. A mathematician does not know how to describe it. This reflects the absence of an appropriate language. One has to develop such a language. So, mathematicians should interact with people from other fields, understand their problems, and translate them into mathematical language, much like von Neumann did.

"If a machine can speak as I do, then the language I am using is not very complex."
